Name——————————————, ID ——————

MTH 221, Linear Algebra, Exam I, 2013

Ayman Badawi

QUESTION 1. Find the solution-set to the following system

$$x_2 + x_3 - x_4 = -2$$

$$x_1 - x_2 - x_3 + x_4 = 7$$

$$2x_1 - 2x_2 - 2x_3 + 2x_4 = 14$$

QUESTION 2. For what values of k will the following system be consistent?

$$x_1 + 2x_2 - 8x_3 = 10$$

$$-x_1 - 2x_2 + kx_3 = -9$$

$$2x_1 + 4x_2 + kx_3 = 16$$

QUESTION 3. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ -1 & 4 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & 2 \\ -3 & 2 \\ 2 & -1 \end{bmatrix}$. Let $D = AB$ and $C = BA$

a) Find D_2

b) Find C_3

c) Find c_{21}

QUESTION 4. Let $C = \begin{bmatrix} a & b & a \\ c & d & c \\ e & f & e \end{bmatrix}$ and $K = \begin{bmatrix} 2a \\ 2c \\ 2e \end{bmatrix}$, where $a, b, c, d, e, f \in R$. Consider the system of linear equations $CX = K$

a) Give me two particular solutions to the system

b) Convince me that the system has infinitely many solutions.

QUESTION 5. For what values of a, b will the following system be inconsistent?

$$x_1 + 4x_2 + 4x_3 = a$$

$$-2x_1 - 8x_2 + bx_3 = 6$$

QUESTION 6. a) Let $W = \begin{bmatrix} 1 & 2 & -3 \\ -1 & -1 & 3 \\ 4 & 8 & -11 \end{bmatrix}$. If possible find W^{-1}.

b) Let W as in (a) and find the solution-set for the system $WX = \begin{bmatrix} 1 \\ 0 \\ -2 \end{bmatrix}$

QUESTION 7. USE ROW-OPERATIONS ONLY to find $\begin{bmatrix} 1 & 0 \\ -3 & 1 \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix} \begin{bmatrix} 1 & 2 & 1 \\ -4 & 3 & 0 \end{bmatrix}$

QUESTION 8. Let A be a 3×3 matrix such $A \;\; \overrightarrow{2R_1 + R_3 \to R_3} \;\; B \;\; \overrightarrow{2R_1} \;\; C$

a) Find two elementary matrices E, F such that $EFC = A$

b)Find a 3×3 matrix D such that $DA = C$

c) Find a 3×3 matrix L such that $LB = C$

d) **EXTRA CREDIT 3 points: Name the Arab Idol for the year 2013:**

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MTH 221, Linear Algebra, Exam II, 2013

Ayman Badawi

QUESTION 1. Given A is a 4×4 matrix such that

$$A \xrightarrow{3R_1 + 2R_2 \to R_2} B \xrightarrow{-R_2 + R_4 \to R_4} C \xrightarrow{4R_3} \begin{bmatrix} 4 & 5 & 9 & 2 \\ -4 & -4 & 6 & 2 \\ 0 & 0 & 0 & 4 \\ 0 & 0 & 2 & 0 \end{bmatrix}$$

a) Find $det(A)$.

b) Find the (3, 4)-entry of C^{-1}

c) Find the solution-set for the system of L.E $\quad AX = \begin{bmatrix} 1 \\ -2 \\ 2 \\ 4 \end{bmatrix}$

QUESTION 2. Given A is a 4×4 matrix such that $det(A) = 2013$. Let K be the first row of A. Find the solution-set for the system $A^T X = K^T$. Explain briefly your solution.

QUESTION 3. a) Let $A = \begin{bmatrix} 4 & 7 & a \\ -4 & b & c \\ -8 & -14 & 6 \end{bmatrix}$ and K be the first column of A. For what values of a, b, c will the system $AX = K$ have unique solution? Find the solution-set.

b)Let A as in (a). For what values of a, b, c will the system $AX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ have infinitely many solutions?

QUESTION 4. a) Find a 3×2 matrix A such that

$$A \begin{bmatrix} 4 & -2 \\ -2 & 0 \end{bmatrix} + A = \begin{bmatrix} 1 & 1 \\ -1 & 0 \\ -4 & 1 \end{bmatrix}$$

b) Given A, B are 3×3 matrices such that $det(A) = -3$, $det(B) = 2$. Then

i) $det(A^2B^{-1})$ =

ii) $det(2A^TB) =$

QUESTION 5. Given $A = \begin{bmatrix} 6 & 8 & 2 \\ -6 & -7 & -1 \\ -6 & -8 & -4 \end{bmatrix}$. Use CRAMER rule to find the value of x_3 when solving the system $AX = \begin{bmatrix} 10 \\ -5 \\ -20 \end{bmatrix}$

QUESTION 6. Let $A = \begin{bmatrix} 1 & 2 & 4 \\ 4 & a & b \\ c & 7 & 10 \end{bmatrix}$ such that $det(A) = 70$. Given $B = \begin{bmatrix} 1 & 2 & 4 \\ -3 & a & b \\ c & 7 & 10 \end{bmatrix}$. Find $det(B)$.

QUESTION 7. Let $A = \begin{bmatrix} 0 & 0 & -1 \\ 1 & 0 & -2 \\ 0 & 1 & 1 \end{bmatrix}$

Find all eigenvalues of A, then for each eigenvalue γ of A, find E_γ.

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MTH 221, Linear Algebra, Final Exam, 2013

Ayman Badawi

QUESTION 1. i) Find a matrix A, 3×3, with eigenvalues: 2, 3, and a for some number a such that $(0, 2, 3) \in E_2(A)$ and $(0, -2, -2) \in E_3(A)$. [You do not need to calculate the eigenvalue a

ii) Let A as in (i) above. Find a nonzero matrix Q, 3×4 such that $AQ = 2Q$

QUESTION 2. For what values of a, b, c, d, e, f will the matrix $A = \begin{bmatrix} 2 & a & b & c \\ 0 & 2 & e & d \\ 0 & 0 & 3 & f \\ 0 & 0 & 0 & 6 \end{bmatrix}$ be diagnolizable?

QUESTION 3. ii) If A is diagnolizable, then find an invertible matrix Q and a diagonal matrix D such that $QDQ^{-1} = A$.

iii)Find a nonzero matrix F, 4×2 such that $AF = 3F$

QUESTION 4. i) Give me an example of two matrices, 2×2, say A and B such that A is row-equivalent to B but A and B have different eigenvalues.

ii) Given $A = \begin{bmatrix} 0 & -4 \\ 1 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 0 & -3 \\ 1 & 4 \end{bmatrix}$. Is there an invertible matrix L, 2×2, such that $LAL^{-1} = B$? If YES, then find L. If NO, then briefly explain.

iii) Given $A = \begin{bmatrix} 4 & 0 \\ 0 & 5 \end{bmatrix}$ and $B = \begin{bmatrix} 5 & 0 \\ 0 & 4 \end{bmatrix}$. Find an invertible matrix Q such that $QAQ^{-1} = B$.

iv) Given A is a 3×3 matrix and $B = A - 2I_3$ where $0, 1, 2$ are the eigenvalues of B. Find the eigenvalues of A and calculate $det(A)$

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MTH 221, Linear Algebra, Final Exam, 2013

Ayman Badawi

QUESTION 1. (5 points) i) Find a matrix A, 3×3, with eigenvalues: 2, 3, -9 such that $(0, 2, 3) \in E_2(A)$ and $(0, -2, -2) \in E_3(A)$. [There are infinitely many possibilities for the matrix A, I only ask you to give me one possibility]. For the matrix A you found, what is $E_{-9}(A)$.

ii) **(3 points)** Let A as in (i) above. Find a nonzero matrix Q, 3×4, such that $AQ = 2Q$

QUESTION 2. (5 points) For what values of a, b, c, d, e, f will the matrix $A = \begin{bmatrix} 2 & a & b & c \\ 0 & 2 & e & d \\ 0 & 0 & 3 & f \\ 0 & 0 & 0 & 6 \end{bmatrix}$ be diagnolizable?

QUESTION 3. (Each 3 points) The following are not subspaces. Use examples to show that one of the two conditions for a set to be a subspace is failed.

i)$M = \{A \in R^{4\times 4} \mid Rank(A)$ł$4\}$

ii)$L = \{f(x) \in P_4 \mid f(0)$ is an integer (whole number)$\}$

iii) $F = \{A \in R^{3\times 3} \mid A$ is non-invertible $\}$

iv) $D = \{f(x) \in P_4 \mid f(0) = 0 \text{ or } f(1) = 0\}$

v) $H = \{a,\ 2a + c - 3,\ c) \mid a, c \in R\}$

QUESTION 4. (Each 3 points) The following are subspaces. Find the dimension and find a basis for each one of them.

i) $F = \{f(x) \in P_6 \mid f(1) = f(0) = f(2) = 0\}$

ii) $K = \{f(x) \in P_3 \mid \int_{-1}^{1} f(x)\, dx = 0\}$

iii) $L = \{(a - b,\ 3a - 3b,\ c,\ d) \mid a, b, c, d \in R\}$

iv) $F = span\{x^3 + x - 3,\ \ -6x,\ \ 2x^3 - 10x - 6\}$

(Question 4 continues):

v) $N = row(A)$ where $A = \begin{bmatrix} 1 & -1 & -2 & 3 \\ -1 & 1 & 2 & 1 \\ 2 & -2 & -4 & 6 \end{bmatrix}$

vi) $G = Col(A)$ where A as in v above.

QUESTION 5. (6 points) Let $M = span\{(1, 0, 0, 1),\ (0, 1, 0, 1),\ (0, 0, 1, -1),\ (2, 2, 2, 2)\}$. Find an orthogonal basis for M.

QUESTION 6. (8 points) Find a matrix A, 4×3, such that $A \begin{bmatrix} -1 & -1 & -1 \\ 0 & -1 & -1 \\ 0 & -1 & 0 \end{bmatrix} + 2A = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$.

QUESTION 7. (4 points) Given $\{(2, 0, 4, -1),\ (-4, 0, -8, 3),\ Q_1,\ Q_2\}$ is a basis for R^4. Find the points Q_1, Q_2.

QUESTION 8. (10 points) Let $T : R^4 \rightarrow R^5$ be a linear transformation such that $T(2,0,0,0) = T(0,0,0,4) = (4,8,0,4,0)$ and $T(2,0,1,4) = T(0,1,0,0) = (-2,-6,2,0,-2)$.

i) Find the standard matrix representation of T.

ii) Find a basis for the range of T

iii) Find a basis for the$Ker(T)$

QUESTION 9. (each 3 points points) Let A be a 3×3 matrix such that $A \xrightarrow{3R_2} B \xrightarrow{2R_1+R_2\to R_2} C \xrightarrow{-4R_3} D = \begin{bmatrix}1 & -2 & 2\\ 0 & 1 & -1\\ 0 & 0 & 1\end{bmatrix}$

i) Find $det(A)$

ii) Find $det(2B^{-1}C)$

iii) Find $det(D^9 + 4D^{-1} - 3I_3)$

iv) Find an invertible matrix M such that $MD = B$

v) Find the solution set for the system $AX = \begin{bmatrix}1\\ -2\\ 1\end{bmatrix}$

QUESTION 10. Consider the system of L.E. $\begin{bmatrix} 1 & -1 & 3 \\ -1 & a & b \\ -2 & 2 & b \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$

i) **(4 points)** For what values of a, b will the system have a unique solution?

ii)**(4 points)** For what values of a, b will the system have infinitely many solutions?

iii) **(3 points)** For what values of a, b will the system be inconsistent?

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